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Continuity, Containment, and Coincidence: Leibniz in the History of the Exact Sciences.

Vincenzo De Risi, ed.; *Leibniz and the Structure of Sciences: Modern Perspectives on the History of Logic, Mathematics, and Epistemology*; Springer Nature, Switzerland, 2019, 298 pp., Hard-back, 103,99 €

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Leibniz and the Structure of Sciences is an excellent collection of papers treating Gottfried Wilhelm Leibniz's projects across a range of sciences including logic, mereology, calculus, geometry, and the mathematics of the infinite. Each paper represents a useful contribution to the extant scholarly literature and helpfully points the way for continued inquiry. The volume should be of value to scholars of Leibniz with interests in the 'exact sciences' and the formal dimensions of his thinking, as well as to historians and philosophers concerned with understanding how Leibniz's thinking anticipates later approaches.

The volume is noteworthy in the latter respect because of the way that it situates Leibniz within the larger histories of science and mathematics. While the scholars represented in the volume are sensitive to the need to understand Leibniz's views within their seventeenth-century intellectual contexts, several also adopt a self-consciously anachronistic approach to their subjects. Thus, the volume includes reconstructions of Leibniz's projects that put them into dialogue with the views of later figures including Georg Cantor, Richard Dedekind, George Boole, and C. S. Peirce (this is literally the case with respect to Cantor, as Richard T. W. Arthur's contribution takes the form of an imagined conversation between Leibniz and the later mathematician). These reconstructions show that even when Leibniz's efforts may have failed outright, were hindered by commitments he maintained to traditional views, or otherwise did not themselves bear fruit, they often came tantalizingly close to realizing crucial later theoretical discoveries. Hence, his approaches may be contrasted productively with those of later theoreticians, revealing both historical continuities as well as theoretical and methodological roads not taken. The portrait of Leibniz that thus emerges from the volume that of an intellectual who not only was responsible for some of the most important intellectual innovations of the seventeenth century, but who was also quite often ahead of his time.

The first paper, “Leibniz on the Logic of Conceptual Containment and Coincidence,” by Marko Malink and Anubav Vasudevan, contrasts two different formal logical calculi developed by Leibniz. While one formulation relies on Leibniz’s well known commitment to truth as the containment of the predicate in the subject concept, the second is founded on an alternative notion of conceptual coincidence and “prefigures in many important respects the algebraic approach to logic which rose to prominence in the second half of the nineteenth century through the work of logicians such as Boole and Jevons” (31). While these two approaches appear distinct, the authors argue that we can reconstruct the calculi as “two alternative axiomatizations of one and the same logical theory” (33). The authors claim that while containment remains fundamental, coincidence can sometimes be deployed pragmatically as the more primitive relation. Malink and Vasudevan also append an ‘algebraic semantics for Leibniz’s containment calculi’ to the paper.

Massimo Mugnai’s paper, “Leibniz’s Mereology in the Essays on Logical Calculus of 1686–1690," surveys a number of writings on logical calculi that Leibniz composed in the late 1680s, showing that he theorized and employed many of the elements featuring in present-day mereological research. While the bulk of the paper concerns itself with reconstructions of the details of Leibniz's definitions of concepts such as parthood, wholeness, coincidence, similarity, and homogeneity, Mugnai's analysis results in a lucid solution of the problem of the relationship between monads and their bodies, one of the thorniest topics in Leibniz's metaphysics (see, e.g. Garber 2009). Specifically, Mugnai argues that Leibniz's mereology provides conceptual resources to understands monads are contained within, but are not parts of, their bodies.

Richard T. W. Arthur's contribution, "Leibniz in Cantor's Paradise: A Dialogue on the Actual Infinite," takes the form of a fictional dialogue between Leibniz and Georg Cantor. This dialogue was composed in the year 2000 and while it has circulated informally since then, it has not been previously published. Arthur's dialogue deftly contrasts Leibniz and Cantor's respective views on number and infinity, arguing that Leibniz's position "constitutes a perfectly clear and consistent third alternative in the foundations of mathematics to the usual dichotomy between the potential infinite (Aristotelianism, intuitionism) and the transfinite (Cantor, set theory), and one that avoids the paradoxes of the infinite" (72). Since the composition of this dialogue, Arthur has become one of the leading contemporary Anglophone commentators on Leibniz's metaphysics (Arthur 2018). Arthur's interpretation of Leibniz's is importantly informed by his understanding of Leibniz's conception of actual infinity and his claim that matter is actually infinitely divided; having the dialogue in print will thus provide an important resource for scholars of Leibniz's metaphysics.

Volume editor Vincenzo De Risi has made important contributions to the study of geometry in Leibniz, for instance in connecting Leibniz's attempts to provide a formal foundation for geometry in the *analysis situs* project with his wider philosophy (De Risi 2007). De Risi's "Leibniz on the Continuity of Space" gives an expansive and rich discussion of the history of continuity in general and Leibniz's attempts to provide a geometrical definition of continuity in particular. For De Risi, "[i]t is remarkable… that Leibniz's last and most fully articulated definition of space has several points in common with the modern understanding of metrical *completeness*" associated with Dedekind (113). Nevertheless, the fact is that Leibniz "lived in an age in which the foundations of mathematics were not found in number theory or analysis, but rather in geometry," and so Leibniz's geometrical project "remained an isolated, and soon forgotten, attempt at maintaining continuity between two ages of mathematics" (166).

In "On the Plurality of Spaces in Leibniz," Valérie Debuiche and David Rabouin investigate the question of whether a plurality of different types of space are possible for Leibniz. Discussion of Leibniz's conception of space have often focused on his debate with the Newtonian Samuel Clarke. There, Leibniz's emphasis on space as a relation between bodies suggests the possibility of a plurality of types of spatial structure: different possible worlds featuring different bodies might exhibit different spatial properties. Debuiche and Rabouin point out that this view is in tension with Leibniz's treatment of geometry as an abstract science of space in which we "can specify various properties of geometrical space (such as tridimensionality, homogeneity, isotropy or continuity)" (171). The authors propose potential resolution to thie tension by suggesting that "mathematical truths are absolutely necessary in God's mind, but they subsist therein only as conditionals. When we turn to consider these truths' actualization, it may well be that some conditions are necessary for their existence which are not given in all possible worlds" (199).

Davide Crippa's "One String Attached: Geometrical Exactness in Leibniz's Parisian Manuscripts" examines Leibniz's reception of Descartes's *Geometrie* during his Paris period of 1672–1676 (during which Leibniz studied mathematics intensely and first developed his calculus). While Descartes's innovations provided algebraic tools for working with geometrical curves, Crippa shows that he additionally excluded certain so-called "mechanical" curves from the proper sphere of "geometry." Thus, while Descartes thought he had placed geometry on firm foundations and made it possible to solve all possible geometrical problems, Leibniz maintained that he had erred by too narrowly circumscribing the domain of geometry. Crippa analyzes Leibniz's own approach to curves excluded by Descartes, focusing on Cycloids and Trichoids generated through manipulation of figurative strings attached to curves, arguing that these efforts were part of a program "intended to give a geometrical foundation to [Leibniz's] infinitesimal analysis by extending the limits of Euclidean and Cartesian geometrical constructions, while maintaining the fundamental ideal of exactness that underscored these latter" (249).

Jürgen Jost's "Leibniz and the Calculus of Variations" analyzes Leibniz's solution to the brachistochrone problem, which "asks to connect two points (x_a, y_a) and (x_b, y_b) in $\mathbb{R}^2$ by such a curve that a particle obeying Galileo's law of gravity and moving without friction travels the distance between those points in the shortest possible time" (255–56). This problem played an important role in the development of the calculus of variations, and while solutions were given by Johann and Jacob Bernouilli, as well as Newton, Jost emphasizes on the generality of Leibniz's solution. Jost shows how Leibniz's solution connects to his work on the refraction of light and his potential early formulation of the principle of least action.

In "Teleology and Realism in Leibniz's Philosophy of Science," Nabeel Hamid analyzes Leibniz's natural philosophical methodology in terms of the language of present-day philosophy of science. Hamid argues that Leibniz adopts teleological considerations of ends in order to ground a realist account of mechanical phenomena. On Hamid's view, Leibniz's 'two-realms' of final and efficient causes do not entail the existence of two separate sets of laws governing the world, but rather that "non-causal considerations—teleological, metaphysical, or architectonic, in Leibniz's various locutions — are implicated in any representation of nature that purports to track the truth or approximate truth about its appearances" (294). In short, for Hamid, Leibniz thinks we need to appeal to non-empirically derived metaphysical principles in order to lend coherence to our research and ensure that it provides reliable knowledge of its objects.

The volume's title emphasizes the *structure* of sciences as distinct forms of knowing. As is evident from a number of the entries in the volume, Leibniz placed extraordinary methodological weight on identifying the basic structure and elements of given sciences, for instance in trying to define the most fundamental logical operations or the basic elements of space. However, while each science exhibits its own distinct structure, for Leibniz, they are not isolated forms of knowing. Indeed, as De Risi suggests in his foreward, in reading these essays on Leibniz's approaches to logic, mathematics, space, the calculus, and natural philosophy, "the reader will come to realize that the connections between them are more complicated still and that, for instance, Leibniz's epistemology of geometry had repercussions on his conceptions of whole and part and these latter on his conception of logic itself" (vi). One minor quibble with the volume is that it could have potentially done more to explicitly thematize the unity and harmony underlying Leibniz's various theoretical endeavors. Given its emphasis on Leibniz's anticipation of later theoretical approaches, there arises the further question of how a holistic accounting of the internal connections between Leibniz's own logic, mereology, geometry, infinitesimal analysis, and metaphysics might shed further light on the continuities and discontinuities between his views and our own. This question aside, the volume is a valuable contribution to our understanding of Leibniz's projects in the exact sciences.

References:

Arthur, Richard T. W. 2018. *Monads, Composition, and Force: Ariadnean Threads through Leibniz's Labyrinth.* Oxford: Oxford University Press.

De Risi, Vincenzo. 2007. *Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space*. Basel/Boston: Birkhauser.
Garber, Daniel. 2009. *Body, Substance, Monad*. Oxford: Oxford University Press.